

Meta-stable SUSY breaking model in supergravity

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ABSTRACT: We analyze a supersymmetry (SUSY) breaking model proposed by Intriligator, Seiberg and Shih in a supergravity (SUGRA) framework. This is a simple and natural setup which demands neither extra superpotential interactions nor an additional gauge symmetry. In the SUGRA setup, the $U(1)_R$ symmetry is explicitly broken by the constant term in the superpotential, and pseudo-moduli field naturally takes non-zero vacuum expectation value through a vanishing cosmological constant condition. Sfermions tend to be heavier than gauginos, and the strong-coupling scale is determined once a ratio of sfermion to gaugino masses is fixed.

KEYWORDS: Supersymmetry Breaking, Supersymmetry and Duality, Supersymmetry Phenomenology.

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1. Introduction

Supersymmetry (SUSY) is the most promising candidate beyond the standard model (SM). The SUSY must be broken in our real world so that an investigation of the SUSY breaking mechanism is important. An idea of dynamical SUSY breaking is one of the most attractive scenario [1], which must have the global $U(1)_R$ symmetry [2].¹ However, this R -symmetry should be explicitly broken in order to realize the gaugino masses as well as avoid massless R -axion. And besides, the dynamical SUSY breaking demands complicated chiral gauge theories [3].² If we have a possibility to construct the simplest SUSY breaking model, it would be a non-chiral gauge theory and the SM gauge group is embedded into a subgroup of its flavor symmetry. It is a direct gauge mediation of the SUSY breaking, which can suppress the SUSY flavor changing neutral currents (FCNCs) [5, 6]. However, in order to construct such a model, we must find a dynamical SUSY breaking model in non-chiral gauge theory without massless R -axion.³ This task seems almost impossible due to the theorems in refs. [1, 2].

This situation is drastically changed if we give up an ordinary sense that we are living in the true vacuum. Recently, Intriligator, Seiberg and Shih (ISS) have discovered a meta-stable SUSY breaking vacuum in $\mathcal{N} = 1$ non-chiral SUSY gauge theory in a free magnetic phase [7]. The model has $SU(N_c)$ gauge group with massive N_f fundamental and anti-fundamental chiral-superfields in the range of $N_c < N_f < \frac{3}{2}N_c$ [8]. Since the SUSY

¹A non-generic superpotential can also break SUSY dynamically, which will not be considered in this paper.

²If massless matters are included, non-chiral theories can also break SUSY dynamically [4].

³There is another difficulty of an existence of too many fields with the SM quantum numbers.

breaking vacuum is not a global minimum but a meta-stable vacuum, the existence of the R -symmetry is not necessarily required as well as the theory can be non-chiral. This situation is attractive as long as the meta-stable vacuum is long-lived compared with an age of the universe, and then a lot of researches on the meta-stable SUSY breaking have been done in various aspects [9]–[16]. However, from the view point of phenomenology, there exists still a difficulty for generating the suitable gaugino masses in the ISS model. For this purpose, people have introduced explicit R -symmetry breaking interactions in the superpotential by hand [11, 12] and also an additional gauge symmetry [13]. These extensions of the ISS model seem complicated and artificial.

In this paper we analyze the ISS model in a supergravity (SUGRA) framework.⁴ This is a simple and natural extension which demands neither extra superpotential interactions nor an additional gauge symmetry. In the SUGRA setup, the R -symmetry is explicitly broken by the constant term in the superpotential, and pseudo-moduli field naturally takes non-zero vacuum expectation value (VEV) through a vanishing cosmological constant condition. Sfermions tend to be heavier than gauginos, and the cutoff scale of the magnetic description is determined once a ratio of sfermion to gaugino masses is fixed. We will also show the meta-stable SUSY breaking vacuum can be sufficiently long-lived.

2. ISS model

First let us show a basic idea of the ISS model. The model is described as an $\mathcal{N} = 1$ SUSY $SU(N_f - N_c)$ gauge theory which consists of N_f dual quarks q, \tilde{q} and a gauge singlet M . This is a magnetic dual of SUSY $SU(N_c)$ gauge theory with massive N_f flavors. Superpotential and Kähler potential are given by

$$W = qM\tilde{q} - \text{Tr}[m^2M], \tag{2.1}$$

$$K_0 = \text{Tr}[M^\dagger M] + q^\dagger q + \tilde{q}^\dagger \tilde{q}, \tag{2.2}$$

respectively. Trace is taken in the flavor space. This Kähler potential is a canonical form since this model is an IR free theory. There exists the R -symmetry, where R -charge assignment is $R(M) = 2$ and $R(q) = R(\tilde{q}) = 0$. We decompose M, q, \tilde{q} , and take m as

$$M = \begin{pmatrix} \hat{Y}_{ab} & Z_{aB} \\ \tilde{Z}_{Ab} & \hat{\Phi}_{AB} \end{pmatrix}, \quad q = \begin{pmatrix} \chi_a \\ \rho_A \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{\chi}_a \\ \tilde{\rho}_A \end{pmatrix}, \quad m = \begin{pmatrix} m\delta_{ab} & 0 \\ 0 & \tilde{m}\delta_{AB} \end{pmatrix} \tag{2.3}$$

where $a, b = 1, \dots, N_f - N_c$ and $A, B = 1, \dots, N_c$ are flavor index. Then eqs. (2.1) and (2.2) are rewritten by the components as

$$W = \chi\hat{Y}\tilde{\chi} + \chi Z\tilde{\rho} + \tilde{\chi}\tilde{Z}\rho + \rho\hat{\Phi}\tilde{\rho} - m^2\text{Tr}[\hat{Y}] - \tilde{m}^2\text{Tr}[\hat{\Phi}], \tag{2.4}$$

$$K_0 = \text{Tr} \left[|\hat{Y}|^2 + |Z|^2 + |\tilde{Z}|^2 + |\hat{\Phi}|^2 \right] + |\chi|^2 + |\rho|^2 + |\tilde{\chi}|^2 + |\tilde{\rho}|^2. \tag{2.5}$$

⁴ R -axion obtains a mass in the SUGRA framework [14]. And, the ISS model in the SUGRA setup was also considered in ref. [15, 16].

By using the field redefinitions, we can always take $\rho = \tilde{\rho} = 0$. And, F -flat conditions, $\partial W/\partial\chi = \partial W/\partial\tilde{\chi} = 0$, are satisfied in the direction of $\hat{Y} = Z = \tilde{Z} = 0$. Then the remaining F -flatness conditions are given by

$$\frac{\partial W}{\partial\hat{Y}} = \chi\tilde{\chi} - m^2\delta_{ab}, \quad \frac{\partial W}{\partial\hat{\Phi}} = -\tilde{m}^2\delta_{AB}, \quad (2.6)$$

which show that the minimum exists at $\chi\tilde{\chi} = m^2\delta_{ab}$. Since the trace part of $\hat{\Phi}$, which we denote $\Phi \equiv \frac{1}{N_c}\text{Tr}[\hat{\Phi}]$, has non-zero F -term \tilde{m}^2 , the SUSY is spontaneously broken. The Φ is a pseudo-moduli field whose fermionic component is the Nambu-Goldstone (NG) fermion of the spontaneous SUSY breaking. We should notice that the traceless part of $\hat{\Phi}$, which we denote $\hat{\Phi}_0 \equiv \hat{\Phi} - \Phi$, has no F -term due to the absence of tadpole term. In this vacuum, the gauge symmetry $SU(N_f - N_c)$ is completely broken, and the flavor symmetry is reduced to $SU(N_f - N_c) \times SU(N_c) \times U(1)_B$.

$\hat{\Phi}$ and $\chi - \tilde{\chi}$ remain massless at the tree level while other fields obtain masses of $\mathcal{O}(m)$.⁵ This means that the VEV of $\hat{\Phi}$ cannot be determined in the tree level. Since the gaugino masses are never generated unless both the SUSY and R -symmetry are broken, we need $\Phi \neq 0$. However, even if $\Phi \neq 0$ and non-zero gaugino masses are generated through the quantum corrections, the R -symmetry is spontaneously broken which induces an unwanted massless R -axion. Therefore, we need an explicit R -symmetry breaking. What is the most natural setup? The answer might be the SUGRA, in which Φ obtains a non-zero VEV as will be shown later.⁶ In fact the ISS model has already implied the existence of SUGRA, since the massless NG fermion should be absorbed into the longitudinal mode of the gravitino. The R -symmetry is explicitly broken in the SUGRA framework through the constant term of the superpotential, which plays a crucial role for realizing the vanishing cosmological constant. We will show that this setup demands neither extra superpotential interactions nor an additional gauge symmetry differently from models so far [11–13].

3. ISS model in SUGRA

Let us now consider the ISS model in the SUGRA setup. We also introduce the next leading order of the Kähler potential⁷

$$K_1 = -\lambda \frac{\text{Tr}[(M^\dagger M)^2]}{\Lambda^2} - \lambda_a \frac{(\text{Tr}[M^\dagger M])^2}{\Lambda^2} - \lambda' \left(\frac{(q^\dagger q)^2}{\Lambda^2} + \frac{(\tilde{q}^\dagger \tilde{q})^2}{\Lambda^2} \right), \quad (3.1)$$

⁵ $\chi - \tilde{\chi}$ is a NG superfield from the broken $U(1)_B$, which could be absorbed into the vector super-multiplet by gauging the $U(1)_B$.

⁶Real part of the scalar component of $\chi - \tilde{\chi}$ also obtains a mass (from corrections of higher order Kähler potential) even when $U(1)_B$ is not gauged, where its imaginary part is a NG boson and a fermion component is still massless.

⁷The coefficients of $(q^\dagger q)^2/\Lambda^2$ and $(\tilde{q}^\dagger \tilde{q})^2/\Lambda^2$ can be (of cause) different. In this case the following analyses a little bit change, but which are easily calculated. Here we just take the same coefficient for simplicity.

for the pseudo-moduli not to take a larger VEV than Λ .⁸ Here Λ is the strong-coupling scale of this theory. We assume the negative signs, $\lambda, \lambda' \sim 1$ and $\lambda_a = 0$ ⁹ in eq. (3.1). Then K_1 is written in components as

$$K_1 = -\frac{\lambda}{\Lambda^2} \text{Tr} \left[|\hat{Y}|^4 + |Z|^4 + |\tilde{Z}|^4 + |\hat{\Phi}|^4 + 2 \left(\hat{\Phi}^\dagger \hat{Y}^\dagger Z \tilde{Z} + \hat{\Phi} \hat{Y} Z^\dagger \tilde{Z}^\dagger + (|\hat{Y}|^2 + |\hat{\Phi}|^2)(|\tilde{Z}|^2 + |Z|^2) \right) \right] - \frac{\lambda'}{\Lambda^2} [|\chi|^4 + |\rho|^4 + |\tilde{\chi}|^4 + |\tilde{\rho}|^4 + 2(|\rho|^2|\chi|^2 + |\tilde{\rho}|^2|\tilde{\chi}|^2)]. \quad (3.2)$$

Notice that this contains the term, $\text{Tr}[\hat{\Phi}^\dagger \hat{Y}^\dagger Z \tilde{Z} + \text{h.c.}]$, which is absent in the original ISS model.

The scalar potential in the SUGRA is given by

$$V = e^{K/M_P^2} \left\{ F_i^\dagger K_{ij}^{-1} F_j - \frac{3|W|^2}{M_P^2} \right\}, \quad (3.3)$$

where $M_P = 1/\sqrt{8\pi G} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale, and indices mean derivative. The F -term in the SUGRA is given by

$$F_i^\dagger = -W_i - K_i \frac{W}{M_P^2}. \quad (3.4)$$

Let us search the potential minimum in the direction of trace part of M . Although the traceless part of M might take VEVs at their minimum, which can be taken away by the shift of the origin. This effect is renormalized by redefining couplings in the following potential.¹⁰ The scalar potential in this direction is calculated as

$$V \simeq e^{K/M_P^2} \left\{ \text{Tr} \left[\frac{1}{1 - \frac{4\lambda}{\Lambda^2} |\hat{\Phi}|^2} \left| \tilde{m}^2 \delta_{AB} + \hat{\Phi}^\dagger \left(1 - \frac{2\lambda}{\Lambda^2} |\hat{\Phi}|^2 \right) \frac{W}{M_P^2} \right|^2 + \frac{1}{1 - \frac{4\lambda}{\Lambda^2} |\hat{Y}|^2} \left| \chi \tilde{\chi} - m^2 \delta_{ab} + \hat{Y}^\dagger \left(1 - \frac{2\lambda}{\Lambda^2} |\hat{Y}|^2 \right) \frac{W}{M_P^2} \right|^2 + \frac{1}{1 - \frac{4\lambda'}{\Lambda^2} |\chi|^2} \left| \hat{Y} \tilde{\chi} + \chi^\dagger \left(1 - \frac{2\lambda'}{\Lambda^2} |\chi|^2 \right) \frac{W}{M_P^2} \right|^2 + \frac{1}{1 - \frac{4\lambda'}{\Lambda^2} |\tilde{\chi}|^2} \left| \hat{Y} \chi + \tilde{\chi}^\dagger \left(1 - \frac{2\lambda'}{\Lambda^2} |\tilde{\chi}|^2 \right) \frac{W}{M_P^2} \right|^2 - \frac{3|W|^2}{M_P^2} \right] \right\}. \quad (3.5)$$

The zero-th order $((1/\Lambda)^0$ and $(1/M_P)^0$) of V corresponds to the tree level potential of the ISS model,

$$V_0 = \text{Tr} [|\tilde{m}^2 \delta_{AB}|^2 + |\chi \tilde{\chi} - m^2 \delta_{ab}|^2] + |\hat{Y} \chi|^2 + |\hat{Y} \tilde{\chi}|^2. \quad (3.6)$$

⁸If we do not introduce K_1 , the VEV of pseudo-moduli would be the Planck scale as the usual Polonyi model [17]. The VEV larger than Λ is meaningless in the dual description. The similar analyses (including K_1 in the Polonyi model) had been done in refs. [18].

⁹It is just for simplicity. The case of $\lambda_a \neq 0$ is easily calculated, where interactions $\text{Tr}[|\hat{Y}|^2|\Phi|^2]$, $\text{Tr}[|\tilde{Z}|^2|Z|^2]$ are added and the following discussions are changed a little.

¹⁰This is correct up to $\mathcal{O}(\Lambda^{-2})$ and $\mathcal{O}(M_P^{-2})$. The author would like to thank K. Yoshioka for pointing out it.

This determines the vacuum at

$$\chi\tilde{\chi} = m^2\delta_{ab}, \quad \hat{Y} = 0. \quad (3.7)$$

The D -flat condition shows $|\chi| = |\tilde{\chi}|$. Notice again that pseudo-moduli Φ is not determined at the tree level.

The next order of non-zero V is $(1/\Lambda)^2$ and $(1/M_P)^2$. By taking D -flat conditions with assuming real VEVs of the fields, stationary conditions of $\hat{\Phi}$ and \hat{Y} show

$$0 = \frac{dV}{d\hat{\Phi}} \simeq \frac{2\lambda\tilde{m}^4\hat{\Phi}}{\Lambda^2} - \frac{m^2\tilde{m}^2\text{Tr}[\hat{Y}] - c\tilde{m}^2}{M_P^2}, \quad (3.8)$$

$$0 = \frac{dV}{d\hat{Y}} \simeq 2m^2\hat{Y} + \frac{8\lambda m^4\hat{Y}}{\Lambda^2} - \frac{2m^2\tilde{m}^2\text{Tr}[\hat{\Phi}] - (4m^4\text{Tr}[\hat{Y}] + \tilde{m}^4\hat{Y}) - 2cm^2}{M_P^2}, \quad (3.9)$$

up to $\mathcal{O}(\Lambda^{-2})$ and $\mathcal{O}(M_P^{-2})$. Here we take the condition of $\hat{\Phi}, \hat{Y} \ll \Lambda$ (neglecting higher order terms of $\hat{\Phi}^n, \hat{Y}^n, \hat{\Phi}^k\hat{Y}^l$ ($n \geq 2, k, l \geq 1$)), and take m, \tilde{m} as real numbers, for simplicity. c is the constant term in the superpotential which is meaningless in the global SUSY theory. This constant c breaks R -symmetry explicitly, which plays a crucial role to realize vanishing cosmological constant. Equations (3.8) and (3.9) suggest that the (local) minimum exists at

$$\Phi \simeq -\frac{c\Lambda^2}{2\lambda\tilde{m}^2M_P^2}, \quad Y \simeq -\frac{c}{M_P^2}, \quad (3.10)$$

where $Y \equiv \frac{1}{N_f - N_c} \text{Tr}[\hat{Y}]$. Thus, Φ is determined and Y is shifted by the SUGRA (and its R -symmetry breaking) effects. Energy scales of the VEVs are surely below the dynamical scale of Λ . The height of the potential at this minimum is given by

$$V_{(\min)} \simeq N_c\tilde{m}^4 - \frac{3c^2}{M_P^2}, \quad (3.11)$$

where we neglect $\mathcal{O}(m^2\tilde{m}^4/M_P^2)$ and $\mathcal{O}(\Lambda^{-2}M_P^{-2})$. Thus, c must be

$$c \simeq \sqrt{\frac{N_c}{3}} \tilde{m}^2 M_P \quad (3.12)$$

in order to realize the vanishing cosmological constant, $V_{(\min)} \simeq 0$.

In summary, the (local) minimum exists at

$$\Phi \simeq -\frac{\sqrt{N_c}\Lambda^2}{2\sqrt{3}\lambda M_P}, \quad Y \simeq -\sqrt{\frac{N_c}{3}} \frac{\tilde{m}^2}{M_P}, \quad \chi = \tilde{\chi} = m, \quad (3.13)$$

$$F_\Phi \simeq \tilde{m}^2, \quad F_Y = 0, \quad F_\chi = F_{\tilde{\chi}} = 0, \quad (3.14)$$

up to $\mathcal{O}(\Lambda^{-2})$ and $\mathcal{O}(M_P^{-1})$.¹¹ The eigenvalues of mass matrix (curvatures at this minimum) of scalar fields, Φ, Y, χ , and $\tilde{\chi}$ are all positive of order

$$\frac{\tilde{m}^4}{\Lambda^2}, \quad m^2, \quad m^2, \quad m^2, \quad (3.15)$$

up to $\mathcal{O}(\Lambda^{-2})$, respectively. Off diagonal elements of the mass matrix are at most $\mathcal{O}(M_P^{-2})$, which can be neglected. This means that the (local) minimum is surely (meta-)stable.

¹¹ $\chi, \tilde{\chi}$ and F_Y have corrections of $\mathcal{O}(M_P^{-2})$, while F_χ and $F_{\tilde{\chi}}$ have only corrections of $\mathcal{O}(M_P^{-4})$.

4. SUSY breaking mediation

In the previous section, a non-zero VEV of pseudo-moduli Φ is naturally obtained in the SUGRA framework. This implies that the gaugino masses can be generated if the SM gauge group is embedded into the unbroken flavor symmetry, $SU(N_c)$ or $SU(N_f - N_c)$. ρ and $\tilde{\rho}$ are identified as messengers, and their mass matrix is given by

$$W \supset (\rho, Z) \begin{pmatrix} \Phi & m \\ m & \left[\frac{\Phi^\dagger Y^\dagger}{\Lambda^2} \right]_{\tilde{F}} \end{pmatrix} \begin{pmatrix} \tilde{\rho} \\ \tilde{Z} \end{pmatrix} \equiv (\rho, Z) \mathcal{M} \begin{pmatrix} \tilde{\rho} \\ \tilde{Z} \end{pmatrix}. \quad (4.1)$$

The gaugino masses are given by

$$M_{\lambda_i} = \frac{\alpha_i}{4\pi} N F_\Phi \frac{\partial}{\partial \Phi} \log \det \mathcal{M} \quad (4.2)$$

where $\alpha_i \equiv g_i^2/(4\pi)$ ($i = SU(3)_c, SU(2)_L, U(1)_Y$), and N is a flavor number of messengers as $N_f - N_c$ (N_c) when the SM gauge group is embedded into $SU(N_c)$ ($SU(N_f - N_c)$).

We should notice that eq. (4.2) can take non-zero values thanks to the Kähler potential K_1 in eq. (3.2), since it contains the interaction, $\Phi^\dagger Y^\dagger Z \tilde{Z}$. Recall that the original ISS model does not have (an R -symmetry breaking) $Z \tilde{Z}$ mass term in the superpotential, then it is difficult to produce gaugino masses. So this situation is expected to be modified in the SUGRA framework. However, unfortunately, the gaugino masses are still too tiny as

$$M_{\lambda_i} \sim N \sqrt{N_c} \frac{\alpha_i}{4\pi} \frac{\tilde{m}^2}{m^2} \frac{\tilde{m}^4}{\Lambda^2 M_P} \sim N \sqrt{N_c} \frac{\alpha_i}{4\pi} \frac{\tilde{m}^2}{m^2} \frac{\tilde{m}^2}{\Lambda^2} m_{3/2}, \quad (4.3)$$

from eqs. (3.13) and (3.14), which have shown $[\Phi^\dagger Y^\dagger / \Lambda^2]_{\tilde{F}} \simeq \sqrt{N_c} \tilde{m}^4 / (\Lambda^2 M_P)$. Equation (4.3) suggests that the gaugino masses induced from gauge mediation are smaller than anomaly mediation effects [19] due to $m, \tilde{m} \ll \Lambda$, where the gravity mediation dominates the gauge mediation.

Although the suitable gaugino masses are not induced from eq. (4.2), we should remember that there are still cubic order contributions ($\mathcal{O}(F_\Phi^3)$) of SUSY breaking [20], which induce the gaugino masses as

$$M_{\lambda_i} \simeq N \frac{\alpha_i}{4\pi} \left(\frac{F_\Phi}{\Phi^2} \right)^2 \frac{F_\Phi}{\Phi} \sim \frac{N}{N_c^2 \sqrt{N_c}} \frac{\alpha_i}{4\pi} \frac{\tilde{m}^6 M_P^5}{\Lambda^{10}}. \quad (4.4)$$

On the other hand, the sfermion masses are generated by the usual two-loop diagrams as

$$m_{\tilde{f}}^2 \simeq N C_i \left(\frac{\alpha_i}{4\pi} \right)^2 \left(\frac{F_\Phi}{\Phi} \right)^2 \sim \frac{N}{N_c} C_i \left(\frac{\alpha_i}{4\pi} \right)^2 \frac{\tilde{m}^4 M_P^2}{\Lambda^4}, \quad (4.5)$$

where C_i s are the quadratic Casimir coefficients, that is, $C_3 = 4/3$, $C_2 = 3/4$, and $C_1 = (3/5)Y^2$. We should notice that eqs. (4.4) and (4.5) are correct only when $\Phi^2 > F_\Phi$ ($\Lambda^4/M_P^2 > \tilde{m}^2$ from eq. (3.13)), so that we cannot take a (global SUSY) limit of $M_P \rightarrow \infty$ where $\Phi \rightarrow 0$. Equations (4.4) and (4.5) suggest that the sfermion masses are heavier than the gaugino masses as

$$\frac{m_{\tilde{f}}}{M_\lambda} \sim N_c^2 \sqrt{\frac{C_i}{N}} \left(\frac{\Lambda^2}{\tilde{m} M_P} \right)^4, \quad (4.6)$$

Fields	fermion mass	scalar mass
$\rho, \tilde{\rho}$	$\Phi \sim 10^7$ GeV	$\Phi \sim 10^7$ GeV
$\tilde{Y}, Z, \tilde{Z}, \chi, \tilde{\chi}$	$m \sim 10^{6.5}$ GeV	$m \sim 10^{6.5}$ GeV
$\hat{\Phi}$	$0.01 \times \frac{F_\Phi}{\Phi} \sim 10^4$ GeV	$0.01 \times \frac{F_\Phi}{\Phi} \sim 10^4$ GeV
gaugino	10^2 GeV	–
sfermion	–	10^4 GeV

Table 1: The mass spectra of the ISS fields in the SUGRA setup. Here we take $m \sim \tilde{m}$, for simplicity.

thus, this model tends to induce the split-SUSY spectrum [21].

Taking the color stability condition $|\tilde{m}| < |\Phi|$ into account, a moderate example of the split-SUSY spectrum might be $m_{\tilde{f}}/M_\lambda \sim 100$, although some tunings of parameters are required for the hierarchy problem. Let us analyze a case of $M_\lambda = \mathcal{O}(100)$ GeV and $m_{\tilde{f}} = \mathcal{O}(10)$ TeV from now on.¹² A rough estimation which neglects N_c, N_f factors and $\mathcal{O}(1)$ coefficients suggests¹³

$$\tilde{m} \sim 10^{6.5} \text{ GeV}, \quad \Lambda \sim 10^{12.5} \text{ GeV} \quad (\Phi \sim 10^7 \text{ GeV}), \quad (4.7)$$

where the magnitude of Φ is really smaller than Λ (and also M_P). Notice that all energy scales are determined once the ratio of sfermion masses to gaugino masses is fixed. In the case of eq. (4.7), we can show that the gravity mediation effects are much smaller than the gauge mediation ones because

$$m_{3/2} \simeq \frac{F_\Phi}{M_P} = \mathcal{O}(10) \text{ keV}. \quad (4.8)$$

Here $m_{3/2}$ is the gravitino mass, which means the gravitino is the lightest superparticle in this model.¹⁴ Anyhow, the FCNCs from the possible Planck suppressed operators in the gravity mediation [24] are negligible.

As for the mass of Φ , the scalar component is estimated as $m^2/\Lambda \sim 3.2$ GeV from eqs. (3.15) while the fermion component is $[F_\Phi^\dagger \Phi^\dagger / \Lambda^2]_{\tilde{F}} \sim 10$ keV from eq. (3.2). However, we should notice that there are one- (two-) loop diagrams¹⁵ which lift up fermion (scalar) masses of both Φ and Φ_0 as 10^4 GeV. Thus, the mass spectra of the ISS fields are summarized in table 1.

We should notice that the strong-coupling scale is about $\Lambda \simeq 10^{12.5}$ GeV in eq. (4.7), which is far below the GUT scale, 2×10^{16} GeV. Thus the model, unfortunately, cannot trace the gauge coupling running. For future reference we will show the QCD renormalization group equation (RGE) of this model in appendix B.

¹²A case of $M_\lambda = \mathcal{O}(100)$ GeV and $m_{\tilde{f}} = \mathcal{O}(1)$ TeV can be also analyzed in the same way, where N_c, N_f factors and $\mathcal{O}(1)$ coefficients must be carefully taken into account.

¹³Hereafter we show absolute values of m, \tilde{m} and VEVs.

¹⁴ $\mathcal{O}(10)$ keV gravitino is free from the gravitino problem, but is difficult to be the warm dark matter which can form the large cosmological structure [22]. Another dark matter, such as axion, might be needed for the large structure.

¹⁵They are similar to the ordinary gauge mediation diagrams, where ρ and $\tilde{\rho}$ propagate in the loops [13].

5. Summary and discussions

We have analyzed the ISS SUSY breaking model in the SUGRA framework. This is a simple and natural setup which demands neither extra superpotential interactions nor an additional gauge symmetry. In the SUGRA setup, the R -symmetry is explicitly broken by the constant term in the superpotential, and pseudo-moduli field naturally takes non-zero VEV through a vanishing cosmological constant condition. Sfermions tend to be heavier than gauginos, and the strong-coupling scale is determined once the ratio of sfermion to gaugino masses is fixed. The meta-stable SUSY breaking vacuum can be sufficiently long-lived as shown in appendix A.

As for the μ -term which also breaks the R -symmetry, it could be derivable in the SUGRA setup through the Giudice-Masiero mechanism [23]. However, it is too small in the parameter set of eq. (4.7), so that we need another mechanism to produce the suitable magnitude of μ -term.¹⁶

Finally, we comment on the small magnitude of eq. (4.2). Remind that this determinant takes non-zero value in the SUGRA setup with the next leading Kähler potential, while it vanishes in the original ISS model. However, unfortunately, it was too small. Can we find another meta-stable vacuum? One candidate is near a singular point of the Kähler potential, $Y \sim \Lambda$. This minimum has nothing to do with the SUGRA effects, and the SUSY mass of $Z\tilde{Z}$ is given by $[\Phi^\dagger Y^\dagger / \Lambda^2]_{\bar{F}} \simeq \tilde{m}^2 / \Lambda$. In this case the gaugino masses become

$$M_{\lambda_i} \sim N \frac{\alpha_i}{4\pi} \frac{\tilde{m}^2}{\Lambda},$$

which are the same order as the sfermion masses. But, this estimation might not be reliable, since the VEV of Y should be smaller than Λ for the correct magnetic description of this theory.

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A. Stability of meta-stable vacuum

Here let us check whether the SUSY breaking meta-stable vacuum (which is found in sections 3 and 4) is long-lived than the age of universe or not. The true vacuum exists at which q, \tilde{q} are decoupled and the gaugino condensation occurs through the pure SUSY $SU(N_f - N_c)$ gauge theory. Neglecting N_c, N_f factors and $\mathcal{O}(1)$ coefficients, the F -flat

¹⁶A simple example of generating μ -term is to introduce a gauge singlet field [25].

conditions and a matching condition between Λ and gaugino condensation scale derive

$$\Phi \simeq m \frac{2(N_f - N_c)}{N_c} \Lambda \frac{3N_c - 2N_f}{N_c}, \quad Y \simeq \frac{\tilde{m}^2}{m \frac{2(2N_c - N_f)}{N_c}} \Lambda \frac{3N_c - 2N_f}{N_c}, \quad q = \tilde{q} = 0. \quad (\text{A.1})$$

Here we neglect small corrections of $\mathcal{O}(M_P^{-1})$, and then the potential height of this vacuum is estimated as

$$V_{\text{SUSY}} \simeq -\tilde{m}^4. \quad (\text{A.2})$$

The distance between the false vacuum and the true vacuum is roughly estimated as

$$\Delta\Phi \simeq \begin{cases} m \frac{2(N_f - N_c)}{N_c} \Lambda \frac{3N_c - 2N_f}{N_c} & (N_c < N_f < 1.46N_c), \\ \frac{\Lambda^2}{M_P} & (1.46N_c < N_f < \frac{3}{2}N_c), \end{cases} \quad (\text{A.3})$$

where we take $m \sim \tilde{m}$ ($\Phi \sim Y$) and eq. (4.7). The potential height of the local maximal¹⁷ is of order

$$V_{\text{peak}} \simeq \tilde{m}^4. \quad (\text{A.4})$$

Then, the bounce action in the triangle approximation is estimated as

$$S \sim \frac{(\Delta\Phi)^4}{V_{\text{peak}}} \sim \begin{cases} \left(\frac{m}{\tilde{m}}\right)^4 \left(\frac{\Lambda}{\tilde{m}}\right)^{\frac{4(3N_c - 2N_f)}{N_c}} & (N_c < N_f < 1.46N_c), \\ \left(\frac{\Lambda^2}{M_P}\right)^4 \frac{1}{\tilde{m}^4} & (1.46N_c < N_f < \frac{3}{2}N_c). \end{cases} \quad (\text{A.5})$$

A lifetime of the meta-stable vacuum is estimated as [13]

$$\tau \sim \frac{1}{\tilde{m}} \left(\frac{1s}{10^{24} \text{ GeV}^{-1}} \right) \sqrt{\frac{2\pi}{S}} e^S, \quad (\text{A.6})$$

which mean that $S \geq 113$ is required for τ to be longer than the age of the universe, $\tau_0 \sim 4.7 \times 10^{17} s$. We can easily show that τ is much longer than τ_0 when $N_f < [\frac{3}{2}N_c]$ (the largest integer smaller than $\frac{3}{2}N_c$) in $N_c < N_f < 1.46N_c$. As for the cases of $1.46N_c < N_f < \frac{3}{2}N_c$ or $N_f = [\frac{3}{2}N_c]$ in $N_c < N_f < 1.46N_c$ ¹⁸, $S = \mathcal{O}(100)$ so that more accurate estimations must be required by taking N_c, N_f factors and $\mathcal{O}(1)$ coefficients into account. Anyhow the meta-stable vacuum can be sufficiently long-lived in the large range of the parameter space.

B. RGE of the model

Let us show the one-loop RGE of the QCD gauge coupling, that is given by

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu') + \frac{b_i}{2\pi} \ln \left(\frac{\mu}{\mu'} \right),$$

¹⁷A local maximum is located at $\chi = \tilde{\chi} = 0$, $\Phi \simeq -\frac{\sqrt{N_c}\Lambda^2}{2\sqrt{3}\lambda M_P}$, and $Y \simeq -\frac{\sqrt{N_c}\tilde{m}^2\Lambda^2}{2\sqrt{3}(\lambda+2\lambda')m^2 M_P}$.

¹⁸The case of $N_c = 11$ and $N_f = 16$ induces the most stringent bound as $S \sim 152$ in $N_c < N_f < 1.46N_c$.

energy scale	b_3 (SU(N_c) \supset SM)	b_3 (SU($N_f - N_c$) \supset SM)
$M_Z < \mu < 10^2$ GeV	$b_3^{\text{SM}} = 7$	$b_3^{\text{SM}} = 7$
10^2 GeV $< \mu < 10^4$ GeV	$b_3^{\text{SM}} - \frac{2}{3} \times 3 = 5$	$b_3^{\text{SM}} - \frac{2}{3} \times 3 = 5$
10^4 GeV $< \mu < 10^{6.5}$ GeV	$b_3^{\text{MSSM}} - b_3^{\hat{\Phi}} = 0$	$b_3^{\text{MSSM}} = 3$
$10^{6.5}$ GeV $< \mu < 10^7$ GeV	$-(N_f - N_c)$	$-N_c - b_3^{\hat{Y}, \hat{\chi}, \hat{\tilde{\chi}}} \times 3 = -N_c - 9$
10^7 GeV $< \mu < \Lambda$	$-2(N_f - N_c)$	$-2N_c - 9$

Table 2: The energy scale dependence of b_3 .

where b_i is one-loop beta function coefficient of the gauge group. The energy scale dependence of b_3 is listed in table 2, where b^{SM} and b^{MSSM} are the QCD one-loop beta function coefficients for the SM and the MSSM, respectively. Taking $\alpha_3(M_Z)^{-1} \sim 8.47$ and b_3 listed above, the QCD coupling at Λ is estimated as

$$\alpha_3(\Lambda)^{-1} \sim 8.47 + \frac{1}{2\pi} \left[7 \ln \left(\frac{M_\lambda}{M_Z} \right) + 5 \ln \left(\frac{m_{\tilde{f}}}{M_\lambda} \right) - 2(N_f - N_c) \ln \left(\frac{\Lambda}{m} \right) \right],$$

when the SM gauge group is embedded into SU(N_c). Here we have neglected the mass difference between $\rho, \tilde{\rho}$ and $Y, Z, \tilde{Z}, \chi, \tilde{\chi}$, and then taken a more stringent bound for the perturbativity. If we require that the QCD coupling constant is perturbative at Λ ($\alpha_3(\Lambda) < 1$), the constraint for the flavor number of messengers ($N_f - N_c$) should be $(N_f - N_c) < 2.4$. As for the SM gauge group is embedded into SU($N_f - N_c$), the b_3 is estimated as

$$\alpha_3(\Lambda)^{-1} \sim 8.47 + \frac{1}{2\pi} \left[7 \ln \left(\frac{M_\lambda}{M_Z} \right) + 5 \ln \left(\frac{m_{\tilde{f}}}{M_\lambda} \right) + 3 \ln \left(\frac{m}{m_{\tilde{f}}} \right) - (2N_c + 9) \ln \left(\frac{\Lambda}{m} \right) \right].$$

This shows the QCD coupling blows up soon above m (around 10^8 GeV for $N_c = 11$)¹⁹.

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¹⁹The embedding of the SM gauge group into SU($N_f - N_c$) requires $N_c > 10$ [11].

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